

Turbulence Elasticity: A Key Concept to A Unified Paradigm of L-I-H Transition

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Abstract

We present a theory of turbulence elasticity, which follows from delayed response of drift waves (DWs) to zonal flow (ZF) shears. It is shown that when $|\langle V \rangle'_{ZF}|/\Delta\omega_k > 1$, with $|\langle V \rangle'_{ZF}|$ the ZF shearing rate and $\Delta\omega_k$ the local turbulence decorrelation rate, the ZF evolution equation is converted from a diffusion equation to a telegraph equation. This insight provides a natural framework for understanding temporally periodic ZF structures, e.g., propagation of the ZF/turbulence intensity front. Furthermore, by incorporating the elastic property of the DW-ZF turbulence, we propose a unified paradigm of low-confinement-mode to intermediate-confinement-mode to high-confinement-mode (L-I-H) transitions. Especially, we derive the onset and termination conditions of the limit cycle oscillations(LCOs), i.e., the I-mode. The transition from an unstable L-mode to I-mode is predicted to occur when $\Delta\omega_k < |\langle V \rangle'_{ZF}| < \langle V \rangle'_{cr}$, where $\langle V \rangle'_{cr}$ is a critical flow shearing rate induced by the turbulence elasticity and is derived explicitly. If $|\langle V \rangle'_{E \times B}| > \langle V \rangle'_{cr}$ ($\langle V \rangle_{E \times B}$ is mean $E \times B$ shear flow driven by edge radial electrostatic field), the I-mode will terminate and spiral into the H-mode.

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I. INTRODUCTION

A predictive model of turbulent transport is essential to the ITER physics program for the purpose of calculating fluxes and transition thresholds. A persistent topic in the turbulent transport research is flow-turbulence interaction, which has been recognized crucial to uncover the underlying physics mechanisms of the turbulent transport and the LH transition[1]. Curiously, while zonal shear decorrelation of the large eddies is widely invoked as the physics mechanism underpinning the nearly Gyro-Bohm scaling, LH transition and etc., most reduced models of transport treat zonal flows in a rather desultory fashion, if at all. The essential elements of a zonal flow model for transport calculations are the basic space and time scales, and the constitutive relation between Reynolds stress and zonal shear. In this work, we present fundamental advances in the theory of zonal shear dynamics which provides the essential ingredients mentioned above.

In the first part, we discuss the physical basis of turbulence elasticity, i.e., a *finite* delay time in the response of the DW turbulence to the ZF shearing[2, 3]. The strength of the turbulence elasticity is measured by the delay time and reflects time-history dependence in the flow-turbulence interaction, i.e., it measures the degree of the frequently employed Fickian (momentum)flux-gradient relation breaking. We give a heuristic discussion of the structure of the delay time, and demonstrate its relative importance for different scenarios of flow-turbulence interaction. By solving the coupled equations of the ZF and the momentum flux, we predict the ZF wave phenomenon. This masco-scale wave phenomenon provides a new way to look at radial propagation of the ZF/turbulence intensity fronts.

In the second part, we propose a simple, unified L-I-H transition model via incorporating the time delay effect in the conventional Predator-Prey(PP) feedback system[4]. The I-mode is an intermediate mode during spontaneous L-H transition and it features a stable phase lag between the DW- and the ZF- intensities, i.e., the LCOs[5, 6]. A quantitative understanding of the onset mechanism of the I-phase has been elusive. Since the LCOs often occur in an “intermediate” regime, where the ZF shearing is stronger than the local turbulence decorrelation rate but not sufficiently strong to fully quench the turbulence, it is unavoidable to consider the time delay effect when addressing the flow-turbulence interaction during the L-H transition. In our new PP model, we not only give a clear explanation of the underlying physics mechanism of the I-mode formation, but also quantitatively predict the

onset and termination conditions of the LCOs[8]. We show that if the delay time exceeds a critical value(which also corresponds to a critical $E \times B$ shearing rate), the DW-ZF system will evolve into a stable LCO state, which corresponds to the transition from an unstable L-mode to I-mode. The termination of the LCOs or the I-H transition occurs if the mean $E \times B$ shear flow, driven by edge ion pressure gradient, is large enough to reduce the delay time below its threshold value. Therefore, the delay time is the quantity parameterizing each stage of the L-I-H transition. During the L-H transition, the delay time is essentially determined by the $E \times B$ shearing time, so, equivalently, we can also say that the $E \times B$ shear controls the transition. The LCO is triggered if the turbulence-driven- $E \times B$ -shear(i.e., the ZF shear) exceeds the local turbulence decorrelation rate, but weaker than the critical shear, and the I-H transition occurs if the ion-pressure-driven- $E \times B$ -shear exceeds the critical shear.

II. THE STRUCTURE OF THE DELAY TIME AND PROPAGATION OF ZF/DW INTENSITY FRONT

In current paper, we discuss the structure of the delay time in a heuristic style. For more precise treatment, see refer.[3]. The basic physics determining the structure of the delay time is the turbulent mixing and hence the delay time is equivalent to the mixing time of the DW turbulence. In the wave turbulence picture, the turbulent mixing is put forward via “collisions” between DW packets. In the existence of ZF fields, there are two mechanisms driving the turbulent relaxation: the local DW-DW scattering(measured by $\Delta\omega_k$), which corresponds to the turbulence scrambling, and the nonlocal DW-ZF interaction(measured by $|\langle V \rangle'_{ZF}|$), which corresponds to “refraction” effect of the ZF to the DW packets. The basic mechanism of the DW-ZF collision is that the ZF shearing can change the radial wavenumber of the DW packets, and then change the ray trajectories of the DW packets. Therefore a natural dimensionless parameter, measuring the relative significance of non-local and local interactions, is $|\langle V \rangle'_{ZF}|/\Delta\omega_k$. If $|\langle V \rangle'_{ZF}|/\Delta\omega_k < 1$, the turbulent relaxation is mainly driven by the local DW-DW scatterings, the memory effect in the momentum transport is negligible. So, the delay time is given by

$$\tau \simeq \Delta\omega_k^{-1}. \quad (1)$$

This scenario corresponds to the L-mode, where the turbulence intensity is strong.

If $|\langle V \rangle'_{ZF} / \Delta\omega_k > 1$, the nonlocal interaction exceeds the local interaction. In this scenario, the ZF shearing is the dominant relaxation mechanism and, correspondingly, the delay time is set by the ZF shearing time,

$$\tau \simeq |\langle V \rangle_{ZF}|^{-1} \quad (2)$$

In experiments, this “elastic” regime is the so-called Dimits shift regime[7], which is a near marginally stable regime and features strong $E \times B$ shear flow. In fact, the L-H transition usually starts from a near Dimits shift regime, so any consistent description of the flow-turbulence interaction should incorporate the time delay/memory effect.

An enlightening way to see the time delay effect in the flow-turbulence interaction is by solving the coupled evolution equations of the ZF and the momentum flux. In other words, the dynamics of the momentum flux should be treated at the same foot as the ZF dynamics. For the ZF, its evolution equation follows as

$$\frac{\partial}{\partial t} \langle V \rangle_{ZF} = -\frac{\partial}{\partial x} \Pi - \gamma_d \langle V \rangle_{ZF}, \quad (3)$$

where Π is the turbulent momentum flux, and γ_d is the ZF friction. Because of the time delay effect, the momentum flux becomes dynamical. Focusing on the physics resulted by a finite τ , we write the evolution equation of Π in a simplest, nontrivial form, i.e.,

$$\frac{\partial}{\partial t} \Pi = -\frac{\Pi - (-D\partial_x \langle V \rangle_{ZF})}{\tau}, \quad (4)$$

D is a turbulent diffusion coefficient. Eqn. (4) indicates that the momentum flux “relaxes” to a diffusion form, not transiently, but after a time τ . According Eqn. (4), if the delay time is very short, one has $\Pi \simeq (-D\partial_x \langle V \rangle_{ZF})$ and hence the conventional Fickian flux-gradient relation sets up. Combining Eqns. (3) and (4) yields a Telegraph equation for the ZF evolution

$$(1 + \tau\gamma_d) \frac{\partial}{\partial t} \langle V \rangle_{ZF} = D \frac{\partial^2}{\partial x^2} \langle V \rangle_{ZF} - \tau \frac{\partial^2}{\partial t^2} \langle V \rangle_{ZF} - \gamma_d \langle V \rangle_{ZF}. \quad (5)$$

The 1st on the RHS of Eqn. (5) is the turbulent viscosity term. The 2nd is essentially a new term induced by the finite delay time, and it is a wavy term and converts the conventional diffusive equation of the ZF into a wave equation. By observing the similarity of the 1st and the 2nd term, we call the 2nd as a turbulence elasticity term with τ the “elastic coefficient”, measuring the strength of the turbulence elasticity.

The key new element here is the “transport of stress”, which gives a time delay and so allows a propagating secondary wave(ZF wave) to develop in the DW ensemble. The upshots of this analysis are that a time delay enters and the familiar diffusive Reynolds equation is replaced by telegraph equation for the zonal flow. This new theory naturally predicts radically propagating ZF/turbulence-intensity front, with the propagation velocity scaling as $\sqrt{\tau/D}$.

III. ELASTIC PREDATOR-PREY SYSTEM: A UNIFIED MODEL OF L-I-H TRANSITION

The L-I-H transition occurs when the strength of the $E \times B$ shearing increases significantly, which is a case that nonlocal process tends to dominant over local process. Therefore, any further unified understanding of the transition mechanism requires incorporating the elastic property of the DW turbulence. In other words, one needs a history dependent of DW-ZF coupling in modeling the flow-turbulence interaction. As the turbulence elasticity features delayed response of the DW turbulence to the ZF shearing, it can induce a phase lag between the DW and the ZF. If the phase lag is stabilized, a steady LCO state(i.e., the I-mode) will form. Motivated by this observation, here we propose a generic and simple L-I-H transition model, which is relevant to determining the onset and termination of the I-mode.

The constitute equations of our elastic 2-fields PP model are[8]

$$\frac{\partial}{\partial t} \varepsilon_D(t) = \gamma_l \varepsilon_D(t) - \gamma_{nl} \varepsilon_D^2(t) - \alpha \varepsilon_Z(t - \tau) \varepsilon_D(t), \quad (6)$$

$$\frac{\partial}{\partial t} \varepsilon_Z(t) = -\gamma_d \varepsilon_Z(t) + \alpha \varepsilon_D(t) \varepsilon_Z(t - \tau). \quad (7)$$

$\varepsilon_D(\varepsilon_Z)$ is the energy intensity of the DW(ZF), γ_l is the linear growth rate of the DW, γ_{nl} describes the local coupling between DWs, γ_d is the ZF frictional damping, and α describes the nonlocal coupling between DW and ZF. The sign of the DW-ZF coupling in Eqn. (7) is opposite to that in Eqn. (6), so that energy conservation is guaranteed during DW-ZF interaction. The exact forms of these coefficients are not crucial to the conclusion of this paper, so we simply take them as given parameters. Eqns. (6) and (7) are the simplest, nontrivial version of an *elastic* PP model. The two fixed points of Eqns. (6) and (7) are $(\varepsilon_D, \varepsilon_Z) = \left(\frac{\gamma_l}{\gamma_{nl}}, 0\right)$ and $(\varepsilon_D, \varepsilon_Z) = \left(\frac{\gamma_d}{\alpha}, \frac{\gamma_l}{\alpha} - \frac{\gamma_{nl}\gamma_d}{\alpha^2}\right)$, which correspond to the L-mode and the H-mode, respectively. The use of a $\varepsilon_Z(t - \tau)\varepsilon_D(t)$ -type DW-ZF coupling in Eqn. (6) is

obvious because of the delayed response of the DW turbulence to the ZF shearing, i.e., the evolution of the DW intensity at time t relies on the “distortion” by the ZF at the earlier time, $t - \tau$. Besides energy conservation, the use of the same type DW-ZF coupling in Eqn. (7) is based on the reason: the nonlinear coupling in Eqn. (7) reflects the back-reaction of the DW turbulence to the ZF shearing, which can be thought as an “elastic force”. The elastic force is proportional to the degree of the DW deformation which is then induced by the ZF shearing at earlier time, so we use the $\varepsilon_Z(t - \tau)\varepsilon_D(t)$ -type in Eqn. (7).

According to experiments, the characteristic time scale of the DW/ZF intensity variation is longer than the delay time[6], i.e., $|\partial_t \ln \varepsilon_{D,Z}| \ll \tau^{-1} \simeq |\langle V \rangle_{ZF}|$, and then the history dependent DW-ZF coupling can be approximated as

$$\varepsilon_D(t)\varepsilon_Z(t - \tau) \simeq \varepsilon_D(t)\varepsilon_Z(t) - \tau\varepsilon_D \frac{\partial}{\partial t} \varepsilon_Z. \quad (8)$$

Substituting Eqn. (8) into Eqns. (6) and (7) yields

$$\frac{\partial}{\partial t} \varepsilon_D = \gamma_l \varepsilon_D - \gamma_{nl} \varepsilon_D^2 - \alpha \varepsilon_Z \varepsilon_D - \frac{\alpha \tau \gamma_d - \alpha^2 \tau \varepsilon_D}{1 + \alpha \tau \varepsilon_D} \varepsilon_Z \varepsilon_D, \quad (9)$$

$$\frac{\partial}{\partial t} \varepsilon_Z = \frac{-\gamma_d \varepsilon_Z + \alpha \varepsilon_D \varepsilon_Z}{1 + \alpha \tau \varepsilon_D}. \quad (10)$$

Eqns.(9) and (10) are a tractable nonlinear system that incorporates the time delay effect. They are equivalent to a “projection” of a more realistic system, such as the 3-fields system composed of the evolutions of $\varepsilon_D, \varepsilon_Z$ and the turbulent momentum flux. Here the effect of dynamical evolution of the turbulent momentum flux is ‘modeled’ by a history dependent DW-ZF coupling. However, the reduced model captures the essence(i.e., history dependent DW-ZF coupling) of the time delay effect, and also is more analytically tractable. Here we use the elastic 2-fields PP model as a paradigm to illustrate the critical role of the delay time in L-H transition dynamics. In the strong shearing scenario, the L-mode fixed point is always stable. To explore the stability of the H-mode fixed point, we linearize Eqns.(9) and (10) near the H-mode fixed point, and the corresponding trace of the Jacobian matrix is

$$tr(J_H) = -\frac{\gamma_{nl}\gamma_d}{\alpha} + \frac{\alpha\tau\gamma_d}{1 + \tau\gamma_d} \left(\frac{\gamma_l}{\alpha} - \frac{\gamma_{nl}\gamma_d}{\alpha^2} \right). \quad (11)$$

In Eqn. (11), the delay time makes a positive contribution, and hence tends to destabilize the H-solution. A critical delay time for transition from an unstable H-mode fixed point to

stable one is given by

$$\text{tr}(J_H) = 0 \implies \tau_{cr} = \frac{\gamma_{nl}}{\alpha\gamma_l - \gamma_{nl}\gamma_d}. \quad (12)$$

If $\tau > \tau_{cr}$, both the fixed points in the Eqns. (9) and (10) will become repellers. According to the Poincaré-Bendixson theorem, the trajectory of the phase point in the phase space of ε_Z and ε_D will then be attracted to a closed orbit, i.e., a limit cycle, and the system will enter the I-mode. In this state, the DW turbulence is not fully quenched, but oscillates. Since the delay time scales as $\tau \simeq |\langle V \rangle'_{ZF}|^{-1}$, we can also obtain a critical ZF shearing rate

$$|\langle V \rangle'_{cr}| = \tau_{cr}^{-1} = \frac{\alpha\gamma_l}{\gamma_{nl}} - \gamma_d. \quad (13)$$

The existence of the H-mode fixed point requires $\alpha\gamma_l/\gamma_{nl} - \gamma_d > 0$, so one has $|\langle V \rangle'_{cr}| > 0$, i.e., the reality condition is satisfied. To initiate the I-mode, it requires that the ZF shearing rate not be “too” strong, i.e., $|\langle V \rangle'_{ZF}| < |\langle V \rangle'_{cr}|$. To destabilize the L-mode, it requires the ZF shearing reach a certain level, $|\langle V \rangle'_{ZF}| > \Delta\omega_k$. Combining with these two requirements, one obtains the condition for the L-I transition as

$$\Delta\omega_k < |\langle V \rangle'_{ZF}| < |\langle V \rangle'_{cr}|. \quad (14)$$

It is widely recognized that the mean $E \times B$ shear flow driven by the radial electric field, which is in turn driven by the ion pressure gradient, plays an important role in “locking” the DW-ZF system to the H-mode. In the preceding section, we showed the delay time to be a new parameter, controlling the state of the DW-ZF system. As the injected power is continuously deposited into the plasma, the edge pressure profile will become steeper and hence a stronger mean $E \times B$ shear flow (denoted as $V_{E \times B}$) will be induced. Clearly, the mean $E \times B$ shear will reduce the delay time, so then impacts the transport barrier dynamics. In the I-mode, the reduction of the delay time causes the increase of the LCO frequency. If the ion pressure profile is steepened sufficiently, the mean $E \times B$ shearing rate will exceed the ZF shearing rate, so that the mean $E \times B$ shearing becomes the dominant turbulence decorrelation mechanism and the delay time is then given by $\tau \sim |\langle V \rangle'_{E \times B}|^{-1}$. In other words, $\langle V \rangle'_{E \times B}$ becomes the main “controller” in the later phase of the I-mode. Once

$$|\langle V \rangle'_{E \times B}| > |\langle V \rangle'_{cr}|, \quad (15)$$

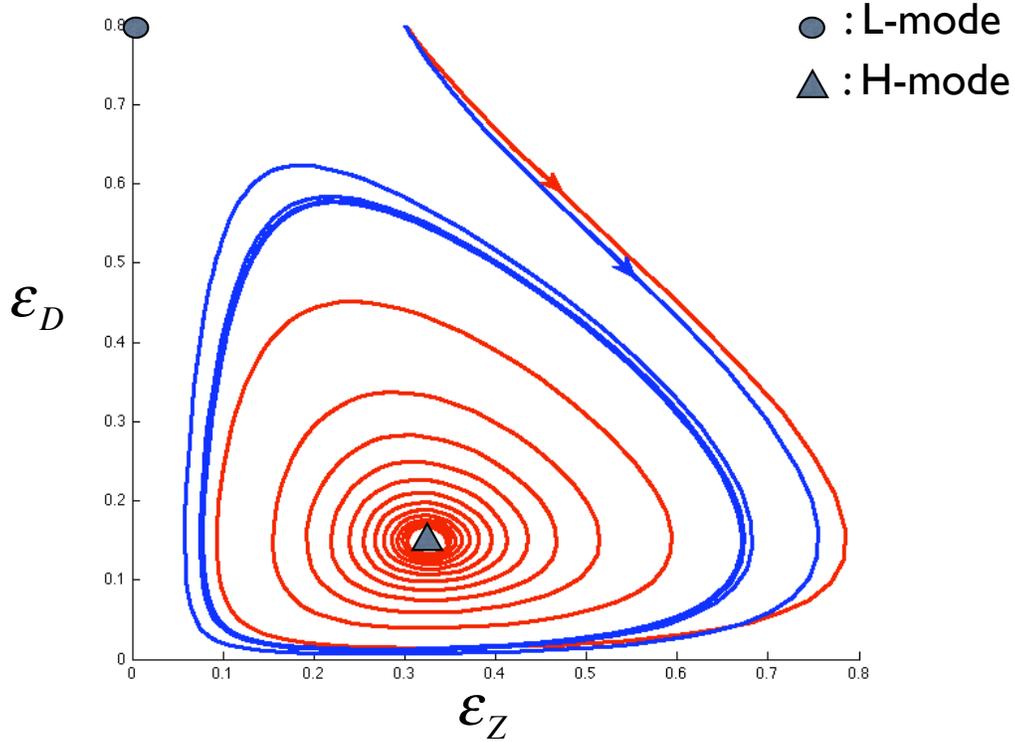


FIG. 1: Blue: I-mode, $\tau > \tau_{cr}$. Red: H-mode, $\tau < \tau_{cr}$. The following parameters are employed in solving Eqns. (9) and (10): $\gamma_l = 0.8$, $\gamma_{nl} = 1$, $\alpha = 2$, $\gamma_d = 0.3$ and initial phase point $(\varepsilon_{D,0}, \varepsilon_{Z,0}) = (0.8, 0.3)$. With these parameters, the H-mode corresponds to $(\varepsilon_Z, \varepsilon_D) = (0.15, 0.325)$ and the L-solution, $(\varepsilon_Z, \varepsilon_D) = (0.8, 0)$. The critical delay time is $\tau_{cr} = 0.77$ with $\tau = 1.8$ in the left figure and $\tau = 0.5$ in the right figure.

the H-solution will become an “attractor”, so that the LCO will terminate and the DW-ZF system will transit from the I-mode to the H-mode(Fig. 1).

Following the above discussion, we arrive at a unified picture of the L-I-H transition(Fig. 2). The ramping injected power can enhance the turbulent Reynolds stress, which then drives stronger ZFs. During this process, the ZF continuously extracts energy from the DW turbulence, and eventually drives the DW-ZF system to the so-called Dimits shift state and initiates the LCO(I-phase). Once the ion pressure profile is steepened sufficiently, the mean $E \times B$ shear reduces the delay time below its threshold value, so as “kills” the LCOs and induces the transition from the I-mode to the H-mode. For strong injection, a strong mean $E \times B$ shear flow can be rapidly generated and hence the L-mode may transit into the

H-mode directly, i.e., the so-called stimulated L-H transition[10].

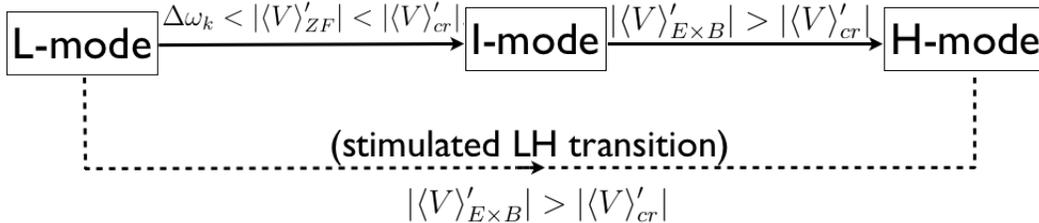


FIG. 2: Sketch of the unified paradigm of L-I-H transition.

IV. SUMMARY AND CONCLUSION

We propose a new concept—turbulence elasticity—in the flow-turbulence interactions. By discussing the structure of the delay time from dynamical first principle, we show the turbulence elasticity is a crucial element of the edge physics in confined plasmas, especially during the transport barrier formation. Since the structure of the delay time can be calculated consistently in a proper framework, e.g., wave kinetic theory, the physics foundation of the turbulence elasticity is solid. The most direct consequence of the turbulence elasticity is the “second sound” phenomenon[9], which indicates the ZF(or turbulence intensity) front can propagate like a wave. For the nonlinear flow-turbulence dynamics, by incorporating the time delay effect we propose a unified, predictive L-I-H transition mode, i.e., the elastic 2-fields PP mode. A very experimental relevant quantity—the critical $E \times B$ shearing rate($|\langle V \rangle'_{cr}|$)—is derived. It is shown that the onset condition of the LCOs(I-mode) is $\Delta\omega_k < |\langle V \rangle'_{ZF}| < |\langle V \rangle'_{cr}|$, and the I-H transition occurs if $|\langle V \rangle'_{E \times B}| > |\langle V \rangle'_{cr}|$.

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